Efficiency Correlation

July 5, 2012

Abstract

In this note we show that even if trigger and muon identification efficiencies (ϵ(id)) are correlated the variation of ϵ(id) produces only loose limits on the variation of the trigger efficiency for events passing the muon-id selection (ϵ(trig|id)).

We also compute the correlations for a Υ(1S) MC generated with Pythia and we find no evidence for strong correlations between the ϵ(id) and ϵ(trig|id).

When computing a systematic uncertainty for ϵ(trig|id) and ϵ(id) we find no strong indication that the two should be varied coherently. On the contrary it would seem more appropriate to vary them independently.

0.1 Introduction

The analysis measures track quality and single muon trigger and identification efficiencies with the tag and probe (T&P) method. Efficiencies are measured sequentially, starting from the muons passing the acceptance the track quality efficiency is first measured. For muons passing the track quality cuts the identification (muon-id) efficiency is measured. Finally, for muons passing also the muon-id cuts the trigger efficiency is measured. The total efficiency can be written as:

$$\epsilon(\text{total}) = \epsilon(\text{trig}|\text{id}) \times \epsilon(\text{id}|\text{track}) \times \epsilon(\text{track}|\text{accepted})\,. \tag{1}$$

The track quality part of the efficiency is very close to 1 and we will neglect it in the following. We will instead focus on $\epsilon(\text{trig}|\text{id})$ and $\epsilon(\text{id})$.

Consider table 1 describing the four possible combinations of the trigger and muon-id selections. Since the total number of events is fixed this system has

A = Pass id - Pass trigger	B = Fail id - Pass trigger
C = Pass id - Fail trigger	D = Fail id - Fail trigger

Table 1: Definition of the identifiers for the four combinations of trigger and muon-id passing and failing cases.

three degrees of freedom. For simplicity, consider the normalization:

$$A + B + C + D = 1\,, \tag{2}$$

it follows that:

$$\begin{aligned}
\epsilon(\text{id}) &= A + C\,, && (3)\\
\epsilon(\text{trigger}) &= A + B\,, && (4)\\
\epsilon(\text{trig}|\text{id}) &= \frac{A}{A+C} = \frac{A}{\epsilon(\text{id})}\,, && (5)\\
\epsilon(\text{trig}|(1-\text{id})) &= \frac{B}{B+D} = \frac{B}{1-\epsilon(\text{id})}\,, && (6)
\end{aligned}$$

where $\epsilon(\text{trig}|(1-\text{id}))$ is the efficiency to pass the trigger for events that do not pass the muon identification.

We can define the correlation between $\epsilon(\text{id})$ and $\epsilon(\text{trigger})$ as:

$$c = \epsilon(\text{trig}|\text{id}) - \epsilon(\text{trig}|(1-\text{id}))\,. \tag{7}$$

With this definition c is -1 for anticorrelated efficiencies ($A = D = 0$), 1 for fully correlated efficiencies ($B = C = 0$) and 0 for no correlation ($\epsilon(\text{trig}|\text{id}) = \epsilon(\text{trig}|(1-\text{id}))$).

All the efficiencies must obviously satisfy the condition $0 \leq \epsilon \leq 1$. This is also true for the relative efficiencies $\epsilon(\text{trig}|\text{id})$ and $\epsilon(\text{trig}|(1-\text{id}))$. An additional constraint is $-1 \leq c \leq 1$.

The system is fully defined by three independent variables. We can use $\epsilon(\text{id})$, $\epsilon(\text{trig}|\text{id})$ and c. $\epsilon(\text{trigger})$ can be written as a function of these variables as:

$$\epsilon(\text{trigger}) = \epsilon(\text{trig}|\text{id})\cdot\epsilon(\text{id}) + \epsilon(\text{trig}|(1-\text{id}))\cdot(1-\epsilon(\text{id})) = \epsilon(\text{trig}|\text{id}) - c\cdot(1-\epsilon(\text{id}))\,. \tag{8}$$

0.2 Limiting Cases

When one of the efficiencies is 1 or 0 or when there is full correlation or anti-correlation the system degenerates to two degrees of freedom.

This can be seen by considering that when, for instance, $\epsilon(\mathrm{id}) = 1$ both B and D are 0. The correlation in this case reduces to $c = \epsilon(\mathrm{trig}|\mathrm{id})$ and this is also equal to $\epsilon(\mathrm{trigger})$. Therefore, in this case the it is sufficient to know either the correlation or $\epsilon(\mathrm{trig}|\mathrm{id})$ to fully define the system.

When the correlation is 1 (-1) the $\epsilon(\mathrm{trigger}) = \epsilon(\mathrm{id})$ ($\epsilon(\mathrm{trigger}) = 1 - \epsilon(\mathrm{id})$) so knowing one of the two again provides a full definition of the system.

Note in particular that for $c = 1$ (full correlation) $\epsilon(\mathrm{trig}|\mathrm{id}) = 1$. If the correlations do not depend on the value of $\epsilon(\mathrm{id})$ then when $\epsilon(\mathrm{id})$ is varied no variation must be made for $\epsilon(\mathrm{trig}|\mathrm{id})$ as it will remain 1.

0.3 Non Limiting Cases and Variations

Outside of the limiting cases $-1 < c < 1$ and $0 < \epsilon < 1$. In this case the system has the full three degrees of freedom, although these constraints make it so that the three variables we are considering are not completely free. Note that the correlation constraint is not linear because it contains ratios.

Before considering how the efficiencies vary given the correlations and to simplify, let us assume that the correlations are constant. By that we mean that even if the muon-id efficiency increases, the trigger efficiency will vary accordingly such that c keeps the same value. Note that $\Delta c = 0$ means $\Delta\epsilon(\mathrm{trig}|\mathrm{id}) = \Delta\epsilon(\mathrm{trig}|(1-\mathrm{id}))$, so $\epsilon(\mathrm{trig}|\mathrm{id})$ is still allowed to change. From equation 8 we have:

$$\Delta\epsilon(\mathrm{trigger}) = \Delta\epsilon(\mathrm{trig}|\mathrm{id}) + c \cdot \Delta\epsilon(\mathrm{id})\,. \tag{9}$$

$\epsilon(\mathrm{trigger})$ will vary more coherently with $\epsilon(\mathrm{id})$ as c increases. However, $\epsilon(\mathrm{trig}|\mathrm{id})$ has still a degree of independence in its variation provided all the efficiency constraints are satisfied.

This is possible because even if the variation of $\epsilon(\mathrm{id})$ and c are known (the first changes to a new number, the second stays the same) as a state of the system is fully determined only by three variables and the constraint of non varying c is not linear, there is still a range of possible values for $\epsilon(\mathrm{trig}|\mathrm{id})$.

0.4 Concrete Example

Considering the following constraints:

$$\begin{aligned}
&0 < \epsilon(\mathrm{trig}|\mathrm{id}) + \Delta\epsilon(\mathrm{trig}|\mathrm{id}) < 1\,,\\
&0 < \epsilon(\mathrm{trig}|(1-\mathrm{id})) + \Delta\epsilon(\mathrm{trig}|(1-\mathrm{id})) < 1 \to 0 < \epsilon(\mathrm{trig}|\mathrm{id}) + \Delta\epsilon(\mathrm{trig}|\mathrm{id}) - c < 1\,,\\
&0 < \epsilon(\mathrm{trig}|\mathrm{id}) + \Delta\epsilon(\mathrm{trig}|\mathrm{id}) + c(\epsilon(\mathrm{id}) + \Delta\epsilon(\mathrm{id})) - c < 1\,,
\end{aligned}$$

where the last disequation is derived from 8. Explicitating the constraint on $\Delta\epsilon(\mathrm{trig}|\mathrm{id})$ they become:

$$\begin{aligned}
&-\epsilon(\mathrm{trig}|\mathrm{id}) < \Delta\epsilon(\mathrm{trig}|\mathrm{id}) < 1 - \epsilon(\mathrm{trig}|\mathrm{id})\,,\\
&c - \epsilon(\mathrm{trig}|\mathrm{id}) < \Delta\epsilon(\mathrm{trig}|\mathrm{id}) < 1 + c - \epsilon(\mathrm{trig}|\mathrm{id})\,,\\
&c(1 - (\epsilon(\mathrm{id}) + \Delta\epsilon(\mathrm{id})) - \epsilon(\mathrm{trig}|\mathrm{id}) < \Delta\epsilon(\mathrm{trig}|\mathrm{id}) < 1 + c(1 - (\epsilon(\mathrm{id}) + \Delta\epsilon(\mathrm{id})) - \epsilon(\mathrm{trig}|\mathrm{id})\,.
\end{aligned}$$

We compute the numbers for table 1 using a realistic Pythia simulation of $\Upsilon(1S)$. The trigger considered is the DoubleMu0. The numbers are reported in table 2. From this table we can compute:

A = 948034	B = 28281
C = 323757	D = 478244

Table 2: Event counts for a $\Upsilon(1S)$ simulation. The trigger considered is the DoubleMu0.

$$\begin{aligned}
\epsilon(\mathrm{id}) &= 0.71517 \pm 0.00034\,,\\
\epsilon(\mathrm{trig}|\mathrm{id}) &= 0.74543 \pm 0.00039\,,\\
\epsilon(\mathrm{trig}|(1-\mathrm{id})) &= 0.05583 \pm 0.00032\,,\\
c &= 0.69045 \pm 0.00050\,,
\end{aligned}$$

where the errors on the efficiencies are binomial. Assuming a 1% variation of $\epsilon(\mathrm{id})$, $\Delta\epsilon(\mathrm{id}) = \epsilon(\mathrm{id}) * 0.01$, and taking the highest lower bound and the lowest higher bound on $\Delta\epsilon(\mathrm{trig}|\mathrm{id})$ we obtain:

$$-0.056 \leq \Delta\epsilon(\mathrm{id}) \leq 0.255\,. \tag{10}$$

This shows how the constraint of non varying correlation poses a small limitation on $\Delta\epsilon(\mathrm{trig}|\mathrm{id})$. While $\epsilon(\mathrm{trigger})$ would be expected to vary as $\epsilon(\mathrm{id})$ when c is high, $\epsilon(\mathrm{trig}|\mathrm{id})$ has a larger degree of independence.

0.5 Correlation of the Trigger Efficiency for Muons Passing the Muon-id Selection with the Muon-id Efficiency

This correlation cannot be evaluated with a single measurement because $\epsilon(\mathrm{trig}|\mathrm{id})$ is computed after $\epsilon(\mathrm{id})$. To evaluate the correlation of these two variables we perform pseudo-experiments and examine the scatter plot of $\epsilon(\mathrm{trig}|\mathrm{id})$ vs $\epsilon(\mathrm{id})$. Using the full Pythia sample we split it into 400 subsamples. The smallest of the 4 cells in the matrix has O(100) events in it. Figure 1 shows a scatter plot of $\epsilon(\mathrm{trig}|\mathrm{id})$ vs $\epsilon(\mathrm{id})$. For comparison 2 shows a scatter plot of $\epsilon(\mathrm{trigger})$ vs $\epsilon(\mathrm{id})$, where a high correlation is expected since c is around 70%. The scale on the vertical axis of the two figures is very similar. Figure 1 does not show a strong correlation between the points especially when compared with figure 2. Figures 3 and 4 show the same for the L1DoubleMuOpen trigger.

Figures from 5 to 7 show the DoubleMu0 trigger efficiency vs the muon identification efficiency in different bins of η for low p_T (3.75 GeV/c $< p_T \leq 5$ GeV/c) muons. Figure 8 shows the same distributions for a higher p_T range (5 GeV/c $< p_T \leq 50$ GeV/c). The ranges are chosen so that the borders match the tag and probe efficiency bins. The limited statistics makes it more difficult to see correlation effects, but it still appears that the $\epsilon(\mathrm{trig}|\mathrm{id})$ is less correlated to $\epsilon(\mathrm{id})$ than $\epsilon(\mathrm{trigger})$ is.

0.6 Conclusions

We considered the effect of correlations between muon-id and trigger efficiency and the effect of their variation assuming constant correlations. If the efficiencies

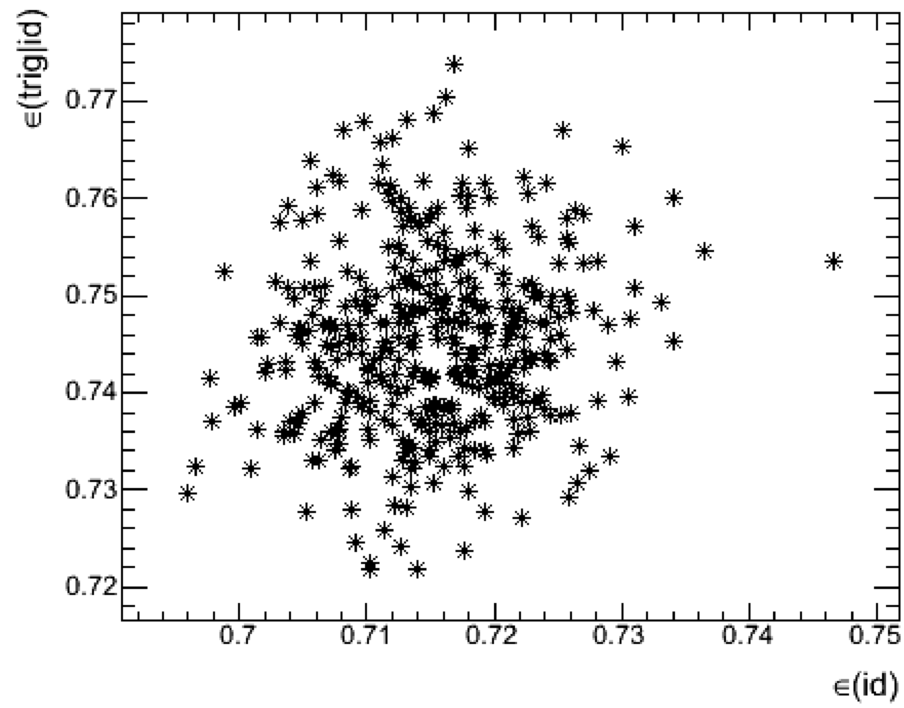


Figure 1: Trigger efficiency for muons passing the identification selection vs muon identification efficiency. The trigger considered is DoubleMu0.

are fully correlated the efficiency to pass the trigger for muons that pass the identification selection should be 1 and its variation ($\Delta\epsilon$(trig|id)) should be 0. For the more generic case assuming constant correlations and varying the muon-id efficiency only poses constraints on $\Delta\epsilon$(trig|id) which for a case close to the one considered in the analysis are expected to be looser than the statistical error on ϵ(trig|id) itself.

The expected variation of $\Delta\epsilon$(trig|id) can be obtained by directly computing its correlation with $\Delta\epsilon$(id). This can be done generating pseudo-experiments and comparing the variation of ϵ(trig|id) and ϵ(id). Comparing the scatter plot of ϵ(trig|id) vs ϵ(id) with the one of ϵ(trigger) vs ϵ(id) the points in the first plot show a much smaller correlation, if any.

When computing a systematic uncertainty for ϵ(trig|id) and ϵ(id) there is no strong indication that the two should be varied coherently. On the contrary it would seem more appropriate to vary them independently.

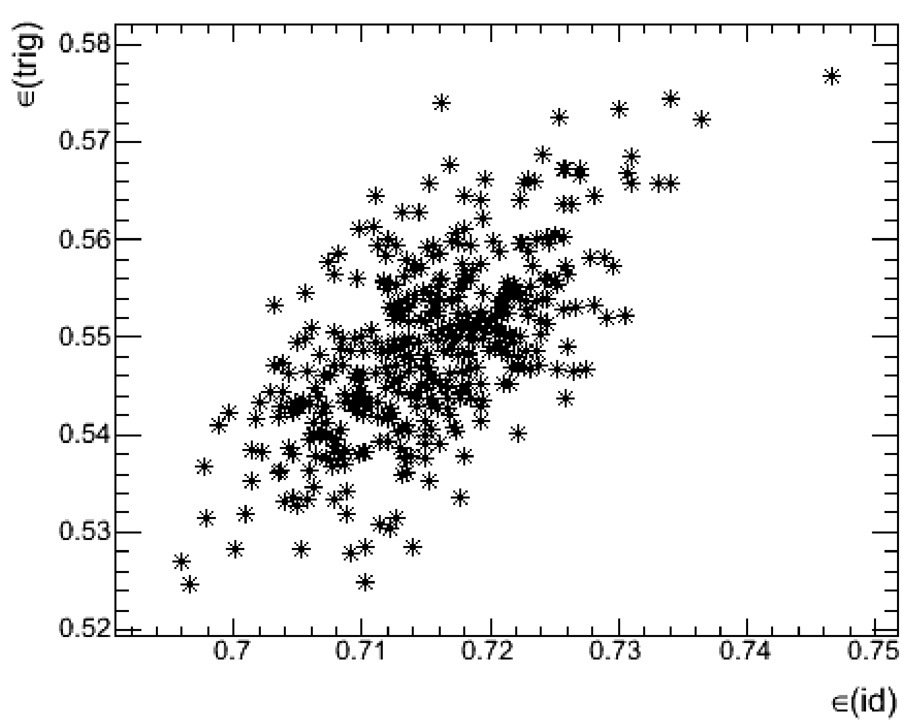


Figure 2: Trigger efficiency vs muon identification efficiency. The trigger considered is DoubleMu0.

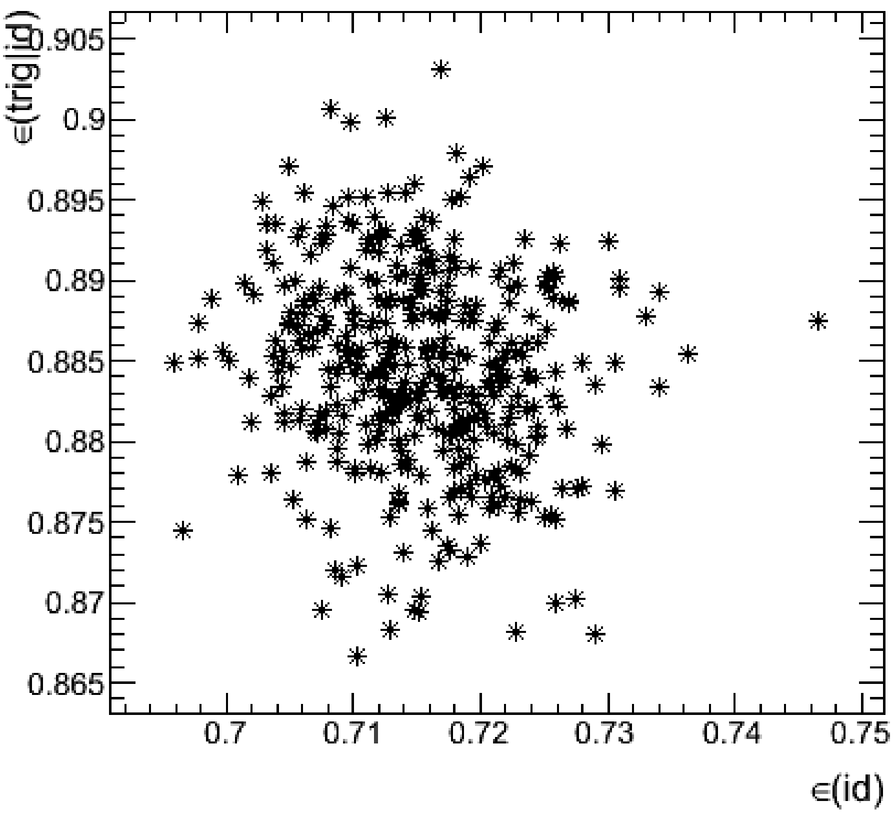


Figure 3: Trigger efficiency for muons passing the identification selection vs muon identification efficiency. The trigger considered is L1DoubleMuOpen.

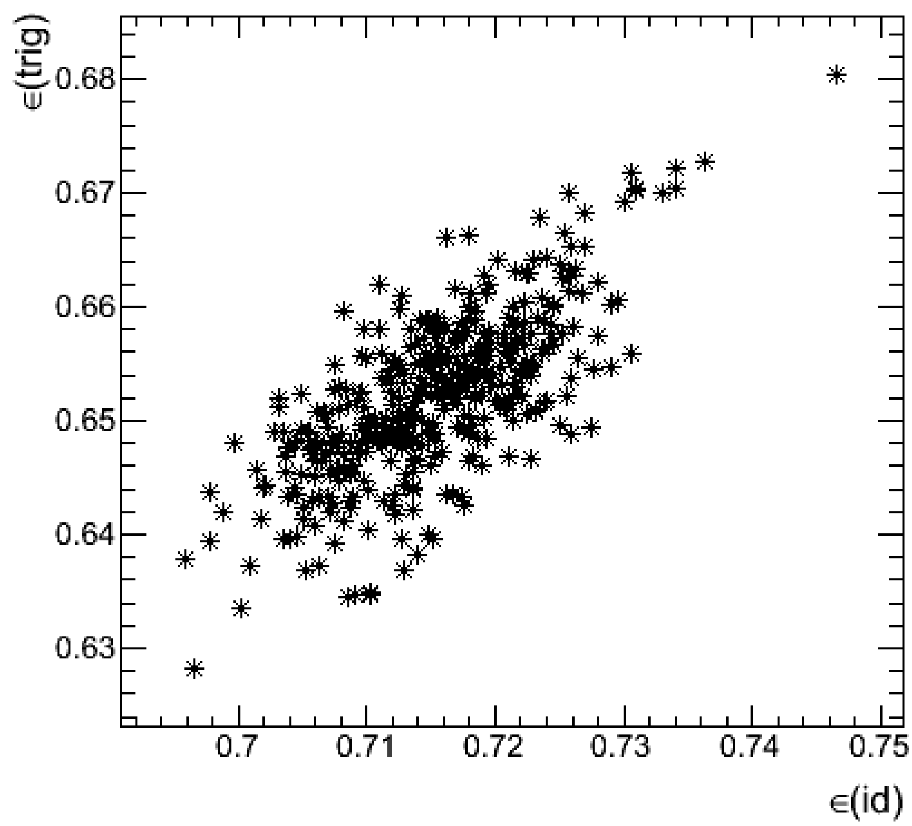
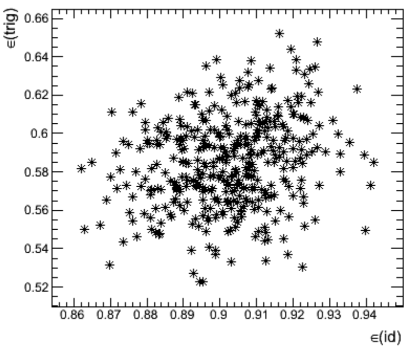
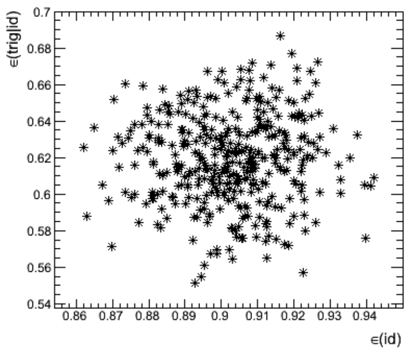


Figure 4: Trigger efficiency vs muon identification efficiency. The trigger considered is L1DoubleMuOpen.

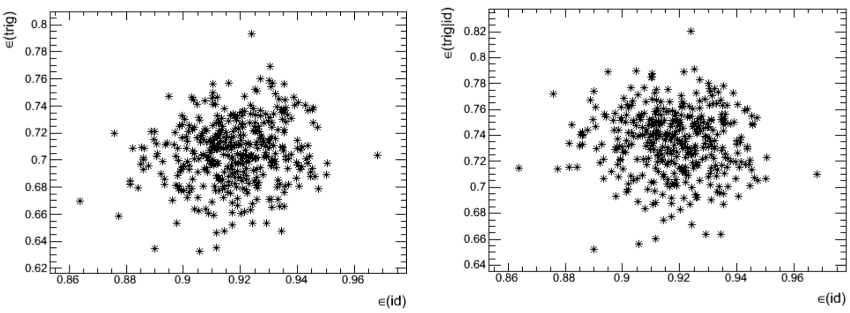


(a)



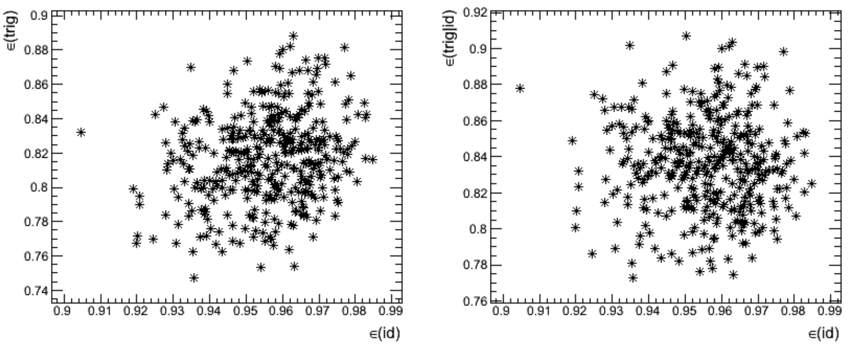
(b)

Figure 5: (a) DoubleMu0 trigger efficiency vs muon-id efficiency for muons with $3.75 < |pt| <= 5$ and $|\eta| <= 0.8$. (b) DoubleMu0 trigger efficiency vs muon-id efficiency for muons with $3.75 < |pt| <= 5$ and $|\eta| <= 0.8$.



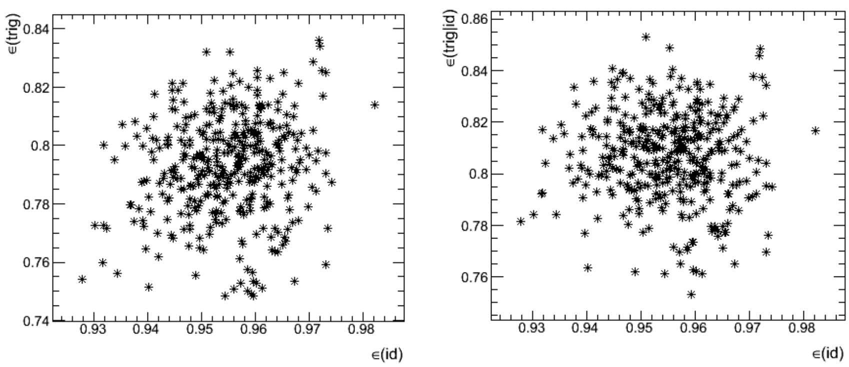
(a) (b)

Figure 6: (a) DoubleMu0 trigger efficiency vs muon-id efficiency for muons with $3.75 < |pt| <= 5$ and $0.8 < |\eta| <= 1.6$. (b) DoubleMu0 trigger efficiency vs muon-id efficiency for muons with $3.75 < |pt| <= 5$ and $0.8 < |\eta| <= 1.6$.



(a) (b)

Figure 7: (a) DoubleMu0 trigger efficiency vs muon-id efficiency for muons with $3.75 < |pt| <= 5$ and $1.6 < |\eta| <= 2.4$. (b) DoubleMu0 trigger efficiency vs muon-id efficiency for muons with $3.75 < |pt| <= 5$ and $1.6 < |\eta| <= 2.4$.



(a) (b)

Figure 8: (a) DoubleMu0 trigger efficiency vs muon-id efficiency for muons with $5 < |pt| <= 50$ and $0.8 < |\eta| <= 1.6$. (b) DoubleMu0 trigger efficiency vs muon-id efficiency for muons with $5 < |pt| <= 50$ and $0.8 < |\eta| <= 1.6$.