Relative Cross Section Fit

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1 Motivation

The likelihood function defined in MuScleFit is:

$$L = \prod_{1}^{N} P(m, \sigma)\,, \tag{1}$$

where m is the reconstructed resonance mass and σ is the estimated resolution on this mass. The value of σ is produced with a suitable function which depends on the kinematics of both muons. N is the total number of resonances at our disposal. Of course events for which we do not reconstruct any resonance do not enter the computation.

We are using Minuit[1] via the TMinuit[2] root interface for the fitting.

The actual function used in the computation is a log likelihood:

$$-2 \log L = -2 \sum_{1}^{N} \log P(m, \sigma)\,. \tag{2}$$

When fitting a single resonance the function $P(m, \sigma)$ is a probability distribution function (*pdf*) defined using the lineshape of the resonance considered. This *pdf* is computed as the value of the convolution of the mass lineshape with a gaussian of mean and sigma defined by m and σ. This is how the fit takes into account the effect of resolution on the mass. Of course the fact that σ is a function of both muons kinematics means that we take into account the full information from both muons. The result of this convolution is normalized as a function of m [1].

When considering more than one resonance in the fit one can sum the *pdf* of each resonance:

$$P = pdf_1 + \ldots + pdf_k\,. \tag{3}$$

This resulting P function is not a pdf anymore because it is not normalized to 1 (but to k in this example). However this does not affect the minimum of (2) because the likelihood function will see this as an additive constant. If we define the normalized P as $P_{norm} = P/k$, then:

$$-2 \log L = -2 \sum_{1}^{N} \log P(m, \sigma) = -2 \sum_{1}^{N} \log k - 2 \sum_{1}^{N} \log P_{norm}(m, \sigma)\,. \tag{4}$$

[1] For example, if the resonance is described by a Lorentzian function, then the convolution results in a Voigtian function normalized as a function of m.

However, Minuit assumes the normalization of the pdf used in the computation of the likelihood to derive the fit errors. The wrong normalization of the pdf function causes a wrong computation of the errors.

Furthermore, when fitting multiple resonances there can be regions where two pdfs give a non-negligible contribution. We can weight each contribution with the probability that a reconstructed mass is one or the other resonance depending on their relative cross sections.

Ultimately, if we weight the pdfs with the relative cross sections we can achieve two goals:

1. Normalization of the P function.
2. Relative weight of the pdfs in overlapping regions.

2 Relative Cross Section Fit

We define the P function as:

$$P = \sum_{i=1}^{k} c_i \times pdf_i \, , c_i = \frac{C_i}{\sum_{i=1}^{k} C_i} \, , \tag{5}$$

where the C_i is the cross section of the i-th resonance. With this definition:

$$\int_0^\infty P(m, \sigma) dm = 1 \, , \sum_{i=0}^{k} c_i = 1 \tag{6}$$

When we fit k resonances we have $k-1$ free parameters to determine. In order to allow minuit to perform the fit using only the $k-1$ free parameters we define a change of variables from the k c_i to $k-1$ y_i. The transformation is defined by:

$$y_0 = 1 \, , y_i = \frac{c_{i+1}}{c_i} \, , i \neq 0 \, . \tag{7}$$

The inverse transformation from the y_i to the c_i is given by

$$c_i = \frac{\prod_{j=0}^{i-1} y_j}{\sum_{k=1}^{n} \prod_{j=0}^{k-1} y_j} \, . \tag{8}$$

See appendix A for more details.

Minuit will return the values of the y_i and by applying the inverse transformation we can derive the relative cross sections and use them to compute the value of P.

3 Application examples

We take a sample of $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ mesons equivalent to $0.1 pb^{-1}$ of integrated luminosity and we perform a fit setting the initial values of the cross sections to 1 for all resonances. Notice that the fit will internally use only the relative cross sections, therfore the actual vales of the cross sections in input are meaningless. Only their relative initial values influence the fit.

The result is shown in figure 1. The input cross sections used are in table 1 while the cross section ratios resulting from the fit are shown in table 2. The results show an excellent agreement with the input values.

The relative cross section fit was checked for stability together with a background, resolution and scale fit in different combinations. No patological behaviours were found.

Resonance	cross section (nb)
$\Upsilon(1S) \to \mu\mu$	13.9
$\Upsilon(2S) \to \mu\mu$	6.33
$\Upsilon(3S) \to \mu\mu$	2.07

Table 1: Cross sections used for the 3 Υ resonances.

Input cross section ratio	Output cross section ratio from the fit
$\Upsilon(1S)/\Upsilon(2S) = 2.20$	2.18 ± 0.15
$\Upsilon(2S)/\Upsilon(3S) = 3.06$	3.11 ± 0.39

Table 2: Relative cross sections as computed from the input cross sections and corresponding values returned by the fit. There is a perfect agreeement within the given errors.

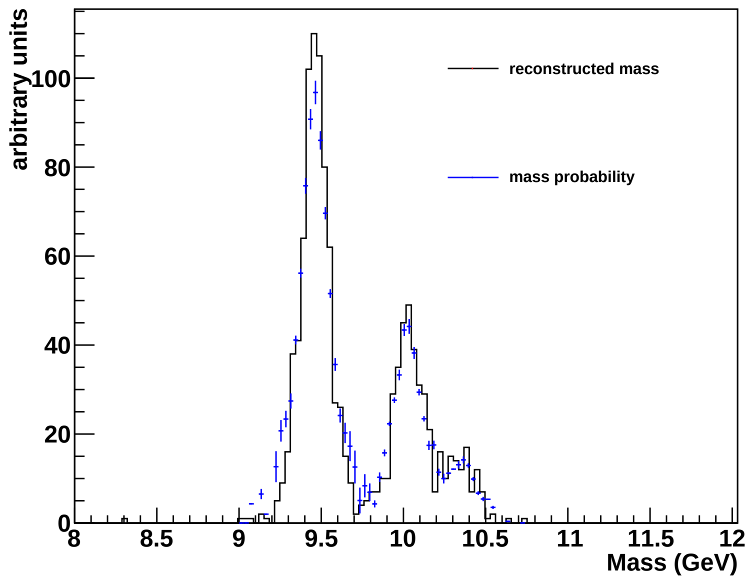

Figure 1: Relative cross section fit with the 3 Υ resonances. The starting values of the cross sections were set to 1 for all the resonances and after the fit the weight is consistently rescaled to the relative cross sections used as input. The remaining differences between the blue and the black curve are attributed mainly to an imperfect modelling of the resolution function.

A Variable transformation

We want to show that the transformation introduced in equation 8 satisfies equation 7 and it is its only solution. From equation 6

$$c_1 = 1 - \sum_{j=1}^{n} c_j \, , \tag{9}$$

while from equation 8

$$c_1 = \frac{1}{\sum_{k=1}^{n} \prod_{j=0}^{k-1} y_j} \, , \tag{10}$$

and

$$1 - \sum_{j=1}^{n} c_j = 1 - \frac{\sum_{j=2}^{n} \sum_{k=0}^{j-1} y_k}{\sum_{k=1}^{n} \prod_{j=0}^{k-1} y_j} \, . \tag{11}$$

By moving the second term of the right side of equation 11 to the left side and recomposing the sum we can readily see that equation 11 is an identity. This proofs that the transformations introduced in equation 7 satisfy equation 6. We should also notice that equation 7 is equivalent to a non omogeneus system of linear equations. This prooves the unicity of the solution.

References

[1] http://lcgapp.cern.ch/project/cls/work-packages/mathlibs/minuit/

[2] http://root.cern.ch/root/html/TMinuit.html