J/Ψ Model

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Abstract

1 Introduction

The MuScleFit [**?**] algorithm performs an unbinned maximum likelihood fit of the data to a reference model in order to calibrate the muon momentum scale and derive the muon momentum resolution. It is essential for the correctness of the calibration that the model is as accurate as possible. For the $J/\Psi \rightarrow \mu_+\mu_-$ the model is built starting from a Summer09 MC sample containing the final state radiation tail. The final state containing muons can see the emission of a photon which takes away part of the energy (final state radiation (FSR)). This results in a lower mass tail in the distribution of the invariant mass from the two muons. The model we use for the decay including the effect of the FSR is **add the model** and the resulting di-muon invariant mass distribution is shown in figure 1.

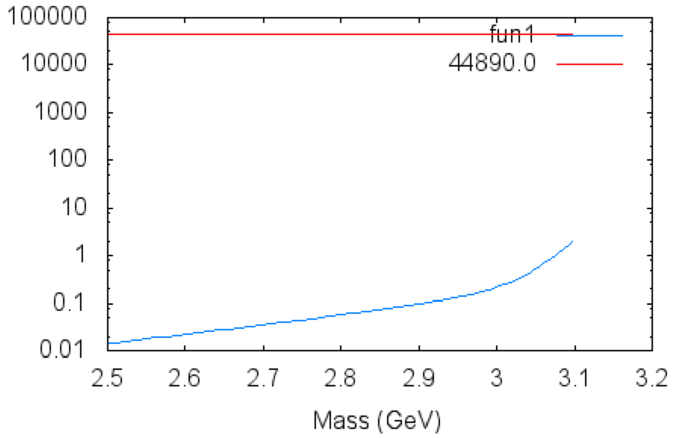

Figure 1: Invariant Mass distribution from model. The blue curve shows FSR tail and the red line shows the heigth of the mass peak which is different from 0 only for $M \in [3.0969, 3.09692]$ (The figure should be improved. Still have to figure out how to do it properly with wxMaxima...).

The mass distribution is described by:

$$f(x) = \begin{cases} a\left(e^{b_1 x + c_1} + e^{b_2 x + c_2} + e^{b_3 x + c_3}\right) & \text{for } 0 \leq x \leq 3.0969 \\ k & \text{for } 3.0969 \leq x \leq 3.09692 \\ 0 & \text{for } x > 3.09692 \text{ and } x < 0 \end{cases}$$

This model is made of a peak 0.2 MeV wide and a tail represented by the sum of three exponential functions. The parameters values are:

$$\begin{aligned} a = & \quad 2.34813 \times 10^{-10} \\ b_1 = & \quad 0.835902 \\ c_1 = & \quad 14.5233 \\ b_2 = & \quad 34.7083 \\ c_2 = & \quad -84.8208 \\ b_3 = & \quad 5.35955 \\ c_3 = & \quad 4.21470 \\ k = & \quad 0.8978/0.00002 \end{aligned}$$

The model must be convoluted with a gaussian function $g(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$ to take into account the effect of the detector resolution. The convolution is defined as

$$(f * g)(x) = \int_{-\infty}^{\infty} f(y) g(x-y) dy \tag{1}$$

By computing explicitly the integral, and taking into account that for any $c \in [a,b]$ is $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, we get

$$(f*g)(x) = \frac{1}{2}\left(k\left[erf\left(\frac{x_{b2}-x}{\sigma\sqrt{2}}\right) - erf\left(\frac{x_b - x}{\sigma\sqrt{2}}\right)\right] - a\sum_{i=0}^{3} f_i(x)(e_{i1}(x) - e_{i2}(x))\right) \tag{2}$$

where $f_i(x)$, $e_{i1}(x)$ and $e_{i2}(x)$ are defined by

$$\begin{aligned} f_i(x) &= e^{\frac{b_i^2\sigma^2}{2} + b_i x + c_i} \\ e_{i1}(x) &= erf\left(\frac{b_i\sigma^2 - x_b + x}{\sigma\sqrt{2}}\right) \\ e_{i2}(x) &= erf\left(\frac{b_i\sigma^2 + x}{\sigma\sqrt{2}}\right) \end{aligned}$$

and $erf(x)$ is the error function defined as $erf(x) = \int_0^x g(z)dz$. We also defined $x_b = 3.0969$ and $x_{b2} = 3.09692$. The result of the integration gives a convoluted distribution where the maximum is shifted with respect to the unsmeared model. The amount of which the maximum is shifted depends on the value of σ. Figure 2 shows the peak when $\sigma = 10^{-6}$, while figures 3, 4, 5 and 6 show the effect when convolving with $\sigma = 0.02, 0.03, 0.04$ and 0.05 respectively. The maximum position is significantly shifted to a lower mass value of ~ 1 MeV with $\sigma = 0.04$ (which is close to the average resolution in data).

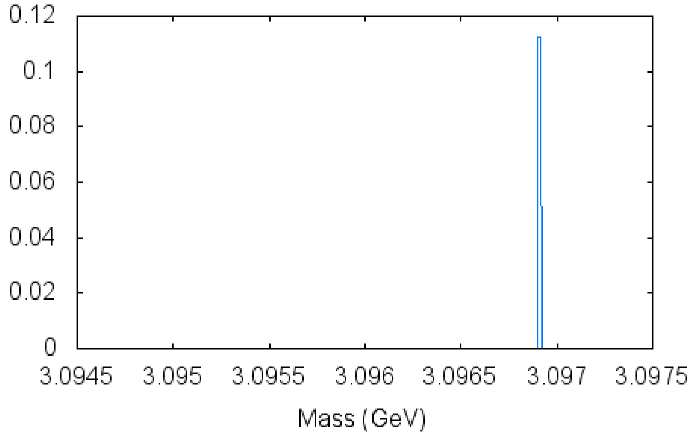

Figure 2: Invariant Mass distribution when the model is convolved with a gaussian of $\sigma = 10^{-6}$. The maximum is essentially in the nominal position.

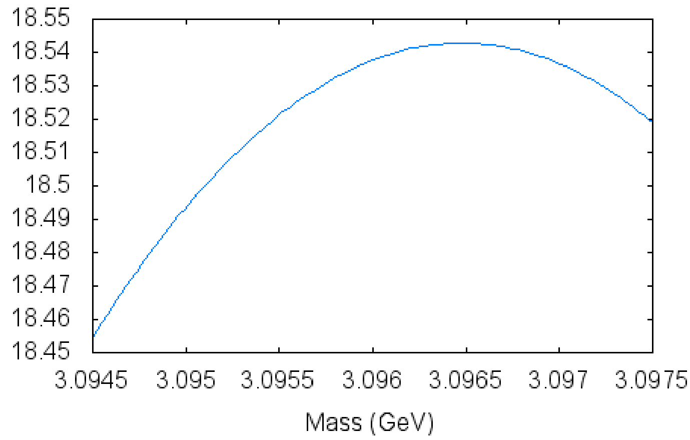

Figure 3: Invariant Mass distribution when the model is convolved with a gaussian of $\sigma = 0.02$.

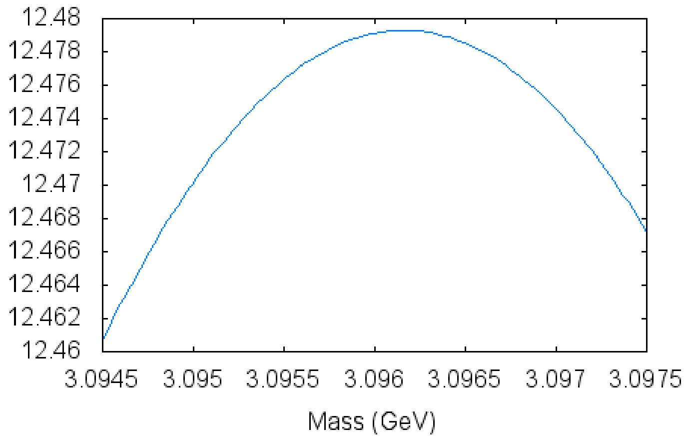

Figure 4: Invariant Mass distribution when the model is convolved with a gaussian of $\sigma = 0.03$.

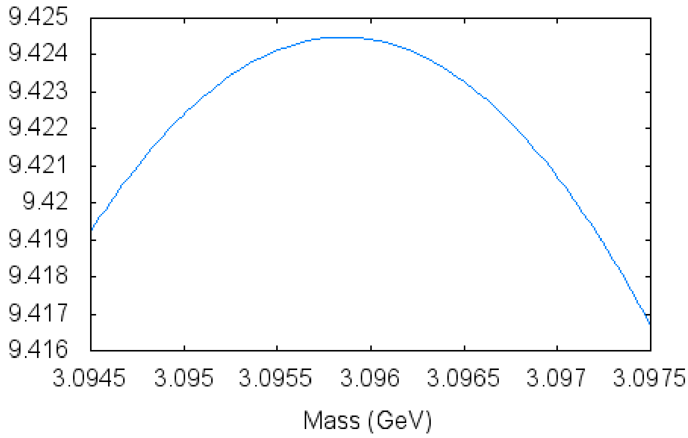

Figure 5: Invariant Mass distribution when the model is convolved with a gaussian of $\sigma = 0.04$.

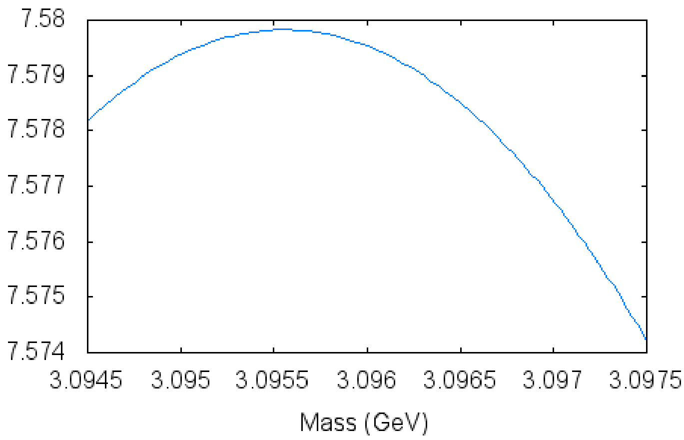

Figure 6: Invariant Mass distribution when the model is convolved with a gaussian of $\sigma = 0.05$.